

A new transmission rate upper bound of HK scheme for Interference system with M-QAM

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Abstract

In this paper, the information transmission rate upper bound of Han and Kobayashi(HK) scheme for real interference system with M-QAM modulation is studied when the system is on weak interference region. A new common bit transmission rate upper bound is derived by a proposed constellation point minimal distance inequality method. For a two-user symmetric interference channel, the different message sending and receiving procedures between strong and weak interference region are taken into considerations in the proposed method, therefore, the new information transmission rate upper bound is more rational than that obtained by direct swapping method in previous paper. Deductions and simulations show that, the new upper bound only differs with the upper bound using direct swapping method on low signal to noise ratio(SNR) region within 1bits/s/Hz, but can achieve two times the upper bound of two-user orthogonal system on high SNR region. Meanwhile, the new upper bound stays the same with different system error probability when the error probability is small, and differs only on low SNR region within 1bits/s/Hz when error probability is large.

Keywords: *HK scheme; Interference elimination; Information transmitting rate; fading channel, transmission rate*

1. Introduction

Interference channel is a common channel model in wireless communication systems. In order to reduce the impact of interference on system performance as well as increasing the system information transmission rate, among different possible methods to mitigate interferences, two main approaches have typically been adopted in literatures. The first is based on the idea of interference avoidance, in which users orthogonalize the communication links in time, frequency or space to reduce the hindering effect of the interference, like TDM [1], OFDM [2] and space time coding [3] et al. The second is interference elimination which focuses on estimating the interference from the received signal and subtracting it from the signal component. The most famous one is HK scheme proposed in 1981[4], which based on jointly typical decoding and successive decoding, splits its message into a common and a private part at each transmitter.

It has aroused wide attention since HK scheme has put forwarded. From the perspective of rate splitting ratio for HK scheme, for a two-user symmetric Gaussian interference channel, an optimal rate splitting ratio for maximizing the sum achievable rate is derived in [5], and a closed-form expression of the corresponding power splitting is presented simultaneously. Since the rate region achieved by HK scheme is the best region known for general interference conditions up to date [6], but the complexity of finding the optimal distribution is high, so the Gaussian interference channel is focused, and the capacity region of the two-user Gaussian interference channel was studied in [7]. Furthermore, a simplified HK scheme is introduced in [8] and the rate region achieved by the simple HK scheme was shown to be within 1 bit/s/Hz of the capacity region. The capacity regions are discussed without the

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consideration of modulations [9-10], however, the real communication system usually adopt high-order modulation to improve the frequency efficient and data transmission rate, so the achievable rate region of HK scheme for communication system with high-order modulation will suffer a loss. And reference [11] gives an information transmission rate upper bound of HK scheme in communication systems with modulation. By simply swapping SNR and interference noise ratio (INR), the upper bound formula on weak interference region can be deduced directly from the upper bound formula on strong interference region. However, since the splitting ratio of HK scheme and the message processing procedures are different between strong and weak interference region, the direct swapping method exits certain irrationality. To this end, a new constellation point minimal distance inequality method is proposed to derive the information transmission rate upper bound for weak interference region. The proposed method follows the decoding order of different information types, and gives the required constellation point minimum distance inequalities for correct decoding. After formula deduction, the information transmission rate upper bound which meets certain system error probability is finally got.

2. Two-user M-QAM interference system model with HK scheme

Fig. 1 shows a two-user M-QAM interference system model with HK scheme in fading channels. The transmitter TX0 and TX1 send their transmitting symbol sequence \mathbf{x}_0 and \mathbf{x}_1 independently. Normalize the transmitting symbol power with $E[|\mathbf{x}_j|^2] \leq 1, j = 0,1$ to save the emission power. After sending the two transmitting symbol sequences by Rayleigh fading channels, the received symbol sequence at both receivers can be written as:

$$\mathbf{y}_i = \sum_{j=0}^1 \mathbf{h}_{ji} \mathbf{x}_j + \mathbf{n}_i \quad (1)$$

where $\mathbf{h}_{ji}, i=0,1$ is the channel fading coefficient of transmitter j to receiver i , \mathbf{n}_i is Gaussian white noise with noise power N_0 .

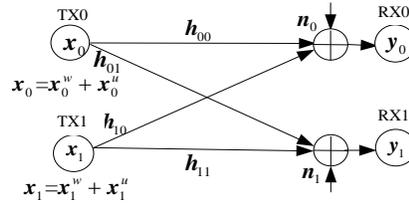


Figure 1. Channel model for two-user M-QAM interference system with HK scheme

From Fig. 1, each transmitter splits their transmitting symbol into two parts, one is common bit symbol \mathbf{x}_j^w , which is decoded at both receivers, and the other is private bit symbol \mathbf{x}_j^u , which is only decoded at the intend receiver. When the private bit symbol comes from the interferer, it will be deemed as noise and will not be decoded. After the common bit symbol of interferer is decoded, remove it from the received symbol completely, then the interference caused by common bit is completely eliminated, the interference when decoding private bit symbol of the intended transmitter is reduced, thereby, improving the interference system performance.

According to the different channel fading coefficients \mathbf{h}_{ji} in Fig. 1, HK scheme defines two interference regions. For the received symbols at the receiver, if the symbol power of the interferer is much more than that of the intend transmitter, i.e., $\mathbf{h}_{00} < \mathbf{h}_{10}$ or $\mathbf{h}_{11} < \mathbf{h}_{01}$, then the interference system is on strong interference region. Since the interference can be completely decoded and removed from the received symbols in this case, all the information bits of interferer are classified into common bits, so there is no private bit with $\mathbf{x}_j = \mathbf{x}_j^w$. The received symbols on strong interference region can be expressed using Eq. (1) as:

$$\mathbf{y}_i = \sum_{j=0}^1 \mathbf{h}_{ji} \mathbf{x}_j^w + \mathbf{n}_i \quad (2)$$

Conversely, if the symbol power of the interferer is lower than that of the intend transmitter, i.e., $\mathbf{h}_{00} > \mathbf{h}_{10}$ or $\mathbf{h}_{11} > \mathbf{h}_{01}$, then the interference system is on weak interference region. In this case, HK scheme splits the transmitting symbols into common bit symbols and private bit symbols for both users, so $\mathbf{x}_j = \mathbf{x}_j^w + \mathbf{x}_j^u$. The received symbols on weak interference region can be expressed using Eq. (1) as:

$$\mathbf{y}_i = \sum_{j=0}^1 \mathbf{h}_{ji} (\mathbf{x}_j^w + \mathbf{x}_j^u) + \mathbf{n}_i \quad (3)$$

Where the common bit symbols \mathbf{x}_j^w and the private bit symbols \mathbf{x}_j^u are subjected to modulation M_j^w -QAM and M_j^u -QAM respectively, so $\mathbf{x}_j^w \in \{M_j^w\text{-QAM}\}$ and $\mathbf{x}_j^u \in \{M_j^u\text{-QAM}\}$. $M_j^w = 2^{R_j^w}$ and $M_j^u = 2^{R_j^u}$ are the corresponding modulation order, R_j^w and R_j^u are the common bit and the private bit transmission rate. Thus, the transmitting symbols of transmitter j satisfies $\mathbf{x}_j \in \{M_j\text{-QAM}\}$, $M_j = M_j^w M_j^u = 2^{R_j^w} 2^{R_j^u} = 2^{R_j}$ is the modulation order of transmitting symbols \mathbf{x}_j , $R_j = R_j^w + R_j^u$ is the sum information transmission rate for the interference system.

From Eq. (2) and (3), $\mathbf{x}_j = \mathbf{x}_j^w$ when the interference system is on strong interference region, and $\mathbf{x}_j = \mathbf{x}_j^w + \mathbf{x}_j^u$ when the interference system is on weak interference region. Obviously, HK scheme takes different division ratio of common bit to private bit between two different interference regions, this will make the interference elimination process of received symbols \mathbf{y}_i different also.

3. Information transmission rate upper bound on weak interference region

As is shown in Eq. (3), on weak interference region, the transmitting symbols of each user are divided into common bit symbols and private bit symbols. Correspondingly, the information transmission rate R_j at the transmitter contains common bit transmission rate R_j^w and private bit transmission rate R_j^u , so $R_j = R_j^w + R_j^u$. Considering the symmetric interference channel model, the common bit and private bit transmission rate upper bound for both users equals to each other respectively, so the user index can be omitted with $R = R^w + R^u$. In order to distinguish the upper bound with other schemes, we note the upper bound obtained in this paper as $R_{HK} = R_{HK}^w + R_{HK}^u$. Since the channel fading factors meet $\mathbf{h}_{00} = \mathbf{h}_{11} = \mathbf{h}_0$ and $\mathbf{h}_{01} = \mathbf{h}_{10} = \mathbf{h}_1$ in symmetric fading channels, note $|\mathbf{h}_0|^2 = P_0$ and $|\mathbf{h}_1|^2 = P_1$. As receiver RX0 and RX1 have the same information processing procedure, we take receiver RX0 for analysis. Therefore, the received symbol sequence of RX0 on weak interference region from Eq. (3) is:

$$\mathbf{y}_0 = \mathbf{h}_0 \mathbf{x}_0^w + \mathbf{h}_0 \mathbf{x}_0^u + \mathbf{h}_1 \mathbf{x}_1^w + \mathbf{h}_1 \mathbf{x}_1^u + \mathbf{n}_0 \quad (4)$$

Based on Eq. (4), the interference elimination process of HK scheme on weak interference region at receiver RX0 goes in three steps. First, decode the common bit symbols of intend transmitter \mathbf{x}_0^w from the received symbols \mathbf{y}_0 , then remove the successful decoded \mathbf{x}_0^w from \mathbf{y}_0 to update the received symbol sequence with $\mathbf{y}_0' = \mathbf{h}_0 \mathbf{x}_0^u + \mathbf{h}_1 \mathbf{x}_1^w + \mathbf{h}_1 \mathbf{x}_1^u + \mathbf{n}_0$. Second, decode the common bit symbols of interferer \mathbf{x}_1^w from the updated received symbol sequence \mathbf{y}_0' , doing the same elimination of \mathbf{x}_1^w from \mathbf{y}_0' , update the received symbols again with $\mathbf{y}_0'' = \mathbf{h}_0 \mathbf{x}_0^u + \mathbf{h}_1 \mathbf{x}_1^u + \mathbf{n}_0$. Finally, decode the private bit symbols of intend transmitter \mathbf{x}_0^u from the renewed received symbols \mathbf{y}_0'' . Thus, the interference elimination of HK scheme on weak interference region completes.

For comparisons, here gives the common bit transmission rate upper bound in literature [11] as $R_{HK, strong}^{(11),w} = \left\lfloor 2 \log \left(1 + \sqrt{INR / SNR} \right) \right\rfloor$ and $R_{HK, weak}^{(11),w} = \left\lfloor 2 \log \left(1 + \sqrt{SNR / INR} \right) \right\rfloor$ for strong and weak interference region respectively. SNR and INR are defined as $SNR = P_0 / N_0$ and $INR = P_1 / N_0$. It can

be seen that, $R_{HK,weak}^{[11],w}$ is obtained by direct swapping SNR and INR in $R_{HK,strong}^{[11],w}$, so we call this method the direct swapping method.

However, the division ratio of common bit to private bit for HK scheme at transmitters and the message processing procedures at receivers are totally different on strong and weak interference region. This direct swapping method ignores the impact of these differences on information transmission rate upper bound, and exits certain irrationality. To solve this problem, this paper proposes a new method to calculate the common bit transmission rate upper bound.

3.1 Common bit transmission rate upper bound

Suppose the receiver can obtain the accurate channel fading coefficients \mathbf{h}_0 and \mathbf{h}_1 by ideal channel estimation. Then, multiply \mathbf{y}_0 with factor \mathbf{h}_0^*/P_0 to get the normalized received symbol $\mathbf{h}_0^*\mathbf{y}_0/P_0 = (\mathbf{x}_0^w + \mathbf{x}_0^u) + (\mathbf{h}_0^*\mathbf{h}_1\mathbf{x}_1^w + \mathbf{h}_0^*\mathbf{h}_1\mathbf{x}_1^u + \mathbf{h}_0^*\mathbf{n}_0)/P_0$. On weak interference region, the modulation constellation with power dissipation factors is shown in Fig. 2.

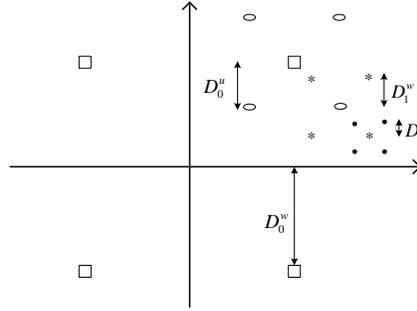


Figure 2. M-QAM modulation with power dissipation factor

Shape "□" and shape "○" represent common bit and private bit constellation points with power dissipation factor $\sqrt{P_0}$ at TX0 respectively, shape "*" and shape "□" represent common bit and private bit constellation points with power dissipation factor $\sqrt{P_1}$ at TX1 respectively. Therefore, the minimum distance between constellation points with power dissipation factor for the four information types $2D_j^w$ and $2D_j^u$ will satisfy $2D_0^w = \sqrt{P_0}d_0^w$, $2D_0^u = (\sqrt{M_0^u} - 1)\sqrt{P_0}d_0^u$, $2D_1^w = (\sqrt{M_1^w} - 1)\sqrt{P_1}d_1^w$ and $2D_1^u = (\sqrt{M_1^u} - 1)\sqrt{P_1}d_1^u$. Among the four formulas, d_j^w and d_j^u are the common bit and private bit constellation point minimum distance without power dissipation factor at transmitter j respectively. Literature [13] shows that, for a given M-QAM modulation with average power P_{M-QAM} , the minimum distance between constellation points without power dissipation factor is $d_{M-QAM} = \sqrt{6P_{M-QAM}/(M-1)}$.

From Fig. 2, in order to decode the common bit symbols of intend transmitter \mathbf{x}_0^w from normalized received symbols $\mathbf{h}_0^*\mathbf{y}_0/P_0$ successfully, the minimum distance with power dissipation factor of \mathbf{x}_0^w should be greater than the sum minimum distance of the other three information types \mathbf{x}_1^w , \mathbf{x}_0^u and \mathbf{x}_1^u , so the four minimum distances with power dissipation factors should satisfy:

$$D_0^w > D_0^u + D_1^w + D_1^u \quad (5)$$

Note the power of common bit and private bit symbols for both transmitters \mathbf{x}_0^w , \mathbf{x}_1^w , \mathbf{x}_0^u and \mathbf{x}_1^u are P_0^w , P_1^w , P_0^u and P_1^u respectively. In symmetric channel model, apply the same power distribution between common bit and private bit symbols for both transmitters, that is to say $P_0^w = P_1^w = P^w$, $P_0^u = P_1^u = P^u$ with $P^w + P^u = 1$. Thus, the corresponding modulation order satisfies

$M_0^w = M_1^w = M^w$ and $M_0^u = M_1^u = M^u$. Put all the above parameters along with the minimum distance without power dissipation formula d_{M-QAM} into Eq. (5), then:

$$\sqrt{P_0}d_0^w > (\sqrt{M_0^u}-1)\sqrt{P_0}d_0^u + (\sqrt{M_1^w}-1)\sqrt{P_1}d_1^w + (\sqrt{M_1^u}-1)\sqrt{P_1}d_1^u \quad (6)$$

Omit the user subscript:

$$\left[\sqrt{P_0} - (\sqrt{M^w}-1)\sqrt{P_1} \right] d^w > (\sqrt{M^u}-1)(\sqrt{P_0} + \sqrt{P_1})d^u \quad (7)$$

The common bit modulation order of intend transmitter can be expressed by a simplification of Eq. (7) as:

$$M^w < \left\{ \frac{(\sqrt{P_0} + \sqrt{P_1})}{\sqrt{P_1}} \left[1 - \sqrt{\frac{P^u}{P^w}} \frac{(\sqrt{M^u}-1)}{\sqrt{M^u-1}} \sqrt{M^w-1} \right] \right\}^2 \quad (8)$$

This means, for weak interference region, if all the parameters mentioned above satisfy Eq. (8), the common bit symbols of intend transmitter \mathbf{x}_0^w can be successfully decoded from the received symbols \mathbf{y}_0 .

Remove the decoded common bit symbols of intend transmitter \mathbf{x}_0^w from the received symbols \mathbf{y}_0 to obtain $\mathbf{y}_0' = \mathbf{h}_0\mathbf{x}_0^u + \mathbf{h}_1\mathbf{x}_1^w + \mathbf{h}_1\mathbf{x}_1^u + \mathbf{n}_0$. Repeat the same normalization operation, multiply \mathbf{y}_0' with factor \mathbf{h}_1^*/P to get the normalized received symbol sequence $\mathbf{h}_1^*\mathbf{y}_0'/P = \mathbf{x}_1^w + (\mathbf{h}_1^*\mathbf{h}_0\mathbf{x}_0^u + \mathbf{h}_1^*\mathbf{h}_1\mathbf{x}_1^u + \mathbf{h}_1^*\mathbf{n}_0)/P$. Similarly, in order to decode the common bit symbols of interferer \mathbf{x}_1^w from $\mathbf{h}_1^*\mathbf{y}_0'/P$ successfully, the minimum distance with power dissipation factor of common bit symbols \mathbf{x}_1^w should be greater than the sum minimum distance of the other two private bit symbols \mathbf{x}_0^u and \mathbf{x}_1^u , so the three minimum distances with power dissipation factor should satisfy inequality:

$$D_1^w > D_0^u + D_1^u \quad (9)$$

Where \mathbf{x}_1^w , \mathbf{x}_0^u and \mathbf{x}_1^u are the constellation point minimum distances with channel power dissipation of the three information types with equation $2D_1^w = \sqrt{P_1}d_1^w$, $2D_0^u = (\sqrt{M_0^u}-1)\sqrt{P_0}d_0^u$ and $2D_1^u = (\sqrt{M_1^u}-1)\sqrt{P_1}d_1^u$. Put the three distance formulas into Eq. (9), the common bit modulation order of the interferer will be:

$$M^w < 1 + \left\{ \frac{\sqrt{P_1}}{(\sqrt{P_0} + \sqrt{P_1})} \sqrt{\frac{P^w}{P^u}} \frac{\sqrt{M^u-1}}{(\sqrt{M^u}-1)} \right\}^2 \quad (10)$$

With these two steps, the common bit symbol decoding procedure for HK scheme on weak interference region completes. Since both users take the same common bit and private bit power allocation parameter, the modulation order of common bit for two transmitters are also the same, M^w should satisfy Eq. (8) and (10) simultaneously. Note parameter Ω as

$\Omega = \frac{\sqrt{P_1}}{(\sqrt{P_0} + \sqrt{P_1})} \sqrt{\frac{P^w}{P^u}} \frac{\sqrt{M^u-1}}{(\sqrt{M^u}-1)}$, then Eq. (8) and (10) can be simplified as:

$$M^w < \left\{ \frac{(\sqrt{P_0} + \sqrt{P_1})}{\sqrt{P_1}} - \sqrt{M^w-1} / \Omega \right\}^2 \quad (11)$$

$$M^w < 1 + \Omega^2 \quad (12)$$

From Eq. (12), we can get $\sqrt{(M^w - 1)}/\Omega < 1$, put it into Eq. (11), so:

$$M^w < \left\{ \frac{(\sqrt{P_0} + \sqrt{P_1})}{\sqrt{P_1}} - \sqrt{(M^w - 1)}/\Omega \right\}^2 < \left\{ \frac{(\sqrt{P_0} + \sqrt{P_1})}{\sqrt{P_1}} - 1 \right\}^2 = \frac{P_0}{P_1} \quad (13)$$

Thus, the common bit transmission rate of HK scheme on weak interference region is $R^w = \log M^w < \log(P_0/P_1)$, the corresponding common bit transmission rate upper bound is:

$$R_{HK}^w = \lfloor \log(P_0/P_1) \rfloor \quad (14)$$

It can be seen from the above derivations that, for the two-user symmetric M-QAM modulation interference system with HK scheme shown in Fig. 1, a new deduction of common bit transmission rate upper bound on weak interference region is proposed. Specifically, the proposed method follows the decoding order of different information types for both users, and obtains the constellation point minimal distance inequality for different symbol sequences been successfully decoded. By formula deductions, conclude the common bit transmission rate upper bound finally. We note this new method the constellation point minimal distance inequality method.

3.2 Private bit transmission rate upper bound

After the common bit symbol sequence \mathbf{x}_0^w and \mathbf{x}_1^w are correctly decoded, remove them from the original received symbol sequence \mathbf{y}_0 to update it with $\mathbf{y}_0^* = \mathbf{h}_0 \mathbf{x}_0^u + \mathbf{h}_1 \mathbf{x}_1^u + \mathbf{n}_0$, Since the channel fading coefficients satisfied $\mathbf{h}_0 > \mathbf{h}_1$ on weak interference region, regard $\mathbf{h}_1 \mathbf{x}_1^u$ and \mathbf{n}_0 together as channel noise in \mathbf{y}_0^* , literature [11] gives the private bit transmission rate upper bound of HK scheme on weak interference region.

If $P_0/P_1 > M^w$, the private bit transmission rate upper bound is

$$R_{HK}^u = \min \left\{ \left\lfloor \log \left(\frac{1 + \delta_1}{1 - \delta_1} \right)^2 \right\rfloor, \left\lfloor \log \left(1 + \frac{P^u P_0}{(P^u P_1 + N_0) \Gamma(M^u, P_e)} \right) \right\rfloor \right\} \quad (15)$$

where $\delta_1 = \frac{P^w}{P^u} \left(\frac{\sqrt{P_1}}{\sqrt{P_0(M^w - 1)}} \right)^2$, $\Gamma(M, P_e)$ is the ‘‘SNR gap’’ [12]

with $\Gamma(M, P_e) = \left[Q^{-1} \left(\frac{\sqrt{M} P_e}{4(\sqrt{M} - 1)} \right) \right]^2 / 3$ and P_e is the system error probability with

$$P_e = \frac{4\sqrt{M} - 4}{\sqrt{M}} Q \left(\sqrt{\frac{3INR}{M-1}} - (\sqrt{M} - 1) \sqrt{\frac{3SNR}{M-1}} \right).$$

On the contrary, if $P_0/P_1 \leq M^w$, the private bit transmission rate upper bound is

$$R_{HK}^u = \min \left\{ \left\lfloor \log \left(\frac{1 + \delta_2}{1 - \delta_2} \right)^2 \right\rfloor, \left\lfloor \log \left(1 + \frac{P^u P_0}{(P^u P_1 + N_0) \Gamma(M^u, P_e)} \right) \right\rfloor \right\} \quad (16)$$

Where $\delta_2 = \frac{P^w}{P^u} \left(\frac{\sqrt{P_0/P_1} - (\sqrt{M^w} - 1)}{\sqrt{P_0/P_1} \sqrt{M^w} - 1} \right)^2$.

Sum up the two interference elimination processes of the received symbol sequence \mathbf{y}_0 at receiver RX0 mentioned above. For the two-user symmetric M-QAM modulation interference system with HK scheme in Fig. 1, based on the proposed constellation point minimal distance inequality method, the information transmission rate upper bound of HK scheme on weak interference region will be:

$$R_{HK} = R_{HK}^w + R_{HK}^u = \left\lfloor \log \left(\frac{P_0}{P_1} \right) \right\rfloor + \min \left\{ \left\lfloor \log \left(\frac{1 + \delta_k}{1 - \delta_k} \right)^2 \right\rfloor, \left\lfloor \log \left(1 + \frac{P^u P_0}{(P^u P_1 + N_0) \Gamma(M^u, P_e)} \right) \right\rfloor \right\}, k = 1, 2 \quad (17)$$

4. Simulations

4.1 Comparisons of information transmission rate upper bound

In order to compare the information transmission rate upper bound with different methods, simulation in this section fixed the interference system error probability $P_e = 10^{-7}$, set INR equals to 5dB, 10dB and 15dB respectively. Fig. 3 gives the information transmission rate upper bound curve R_{HK} with the proposed constellation point minimal distance inequality method. As a comparison, curve $R_{HK}^{[11]}$ gives the information transmission rate upper bound with direct swapping method in literature [11]. In addition, a third curve R_{ORTH} in Fig. 3 gives the information transmission rate upper bound for two-user orthogonal interference avoidance system. In this orthogonal system, two users use two different time slots to transmit their symbol sequence to avoid the user interference, and the corresponding information transmission rate is $R_{ORTH} = \frac{1}{2} \log \left(1 + \frac{SNR}{\Gamma(M, P_e)} \right)$.

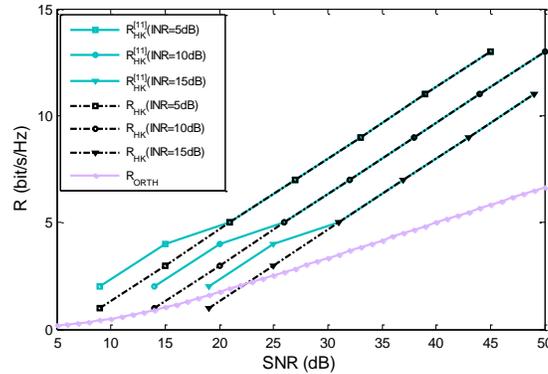


Figure 3. Information transmission rate upper bound curves on weak interference region

From Fig. 3, when $INR = 5dB$ and SNR is greater than 30dB, the proposed information transmission rate upper bound R_{HK} has increased exponentially to that of the orthogonal system R_{ORTH} . However, with the increase of INR, the SNR gain which R_{HK} obtains over R_{ORTH} decreases. This is mainly because, the orthogonal information transmission rate upper bound R_{ORTH} is only affected by SNR, the increase of INR does not change R_{ORTH} value. While for two-user symmetric M-QAM modulation interference system with HK scheme, the increasing of interference will increase the system error probability, and results in the decrease of system information transmission rate upper bound R_{HK} . Therefore, the SNR gain which the proposed information transmission rate upper bound R_{HK} obtains over R_{ORTH} will decrease as INR increasing.

For a comparison of R_{HK} and the upper bound $R_{HK}^{[11]}$ using direct swapping method, Eq. (18) and (19) separately list the common bit transmission rate upper bound $R_{HK}^{[11],w}$ and the total information transmission rate upper bound $R_{HK}^{[11]}$ in [11]:

$$R_{HK}^{[11],w} = \left\lfloor 2 \log \left(1 + \sqrt{P_0 / P_1} \right) \right\rfloor \quad (18)$$

$$R_{HK}^{(11)} = \left[2 \log \left(1 + \sqrt{\frac{P_0}{P_1}} \right) \right] + \min \left\{ \left[\log \left(\frac{1 + \delta_k}{1 - \delta_k} \right)^2 \right], \left[\log \left(1 + \frac{P^u P_0}{(P^u P_1 + N_0) \Gamma(M^u, P_e)} \right) \right] \right\} \quad (19)$$

The difference between R_{HK} in Eq. (17) and $R_{HK}^{(11)}$ in Eq. (19) is mainly reflected on the difference between common bit information transmission rate upper bound R_{HK}^w and $R_{HK}^{(11),w}$, i.e., between Eq. (14) and (18). When $\sqrt{P_0/P_1} = \sqrt{SNR/INR}$ is small, the R_{HK}^w value from Eq. (14) is slightly less than $R_{HK}^{(11),w}$ value calculated by Eq. (18). However, when $\sqrt{P_0/P_1}$ is large, R_{HK}^w and $R_{HK}^{(11),w}$ tend to be equal, thus, the proposed information transmission rate upper bound R_{HK} is tighter than that for $R_{HK}^{(11)}$ only when $\sqrt{P_0/P_1}$ is small, while R_{HK} and $R_{HK}^{(11)}$ tend to be equal when $\sqrt{P_0/P_1}$ is large. Simulation results in Fig. 3 also show that, for a given INR value, when SNR is less than 30dB, which means $\sqrt{P_0/P_1}$ is small, R_{HK} suffers a maximum 1bit/s/Hz rate loss to $R_{HK}^{(11)}$. As SNR increased to more than 30dB, which means $\sqrt{P_0/P_1}$ becomes large, R_{HK} curve and $R_{HK}^{(11)}$ curve coincide with each other completely.

4.2 Impact of P_e on the proposed information transmission rate upper bound R_{HK}

This section discuss the impact of system error probability P_e on the proposed information transmission rate upper bound R_{HK} . Let P_e be 10^{-2} , 10^{-3} and 10^{-4} three large values, R_{HK} curves with three P_e values drawn by Eq. (17) are shown in Fig. 4.

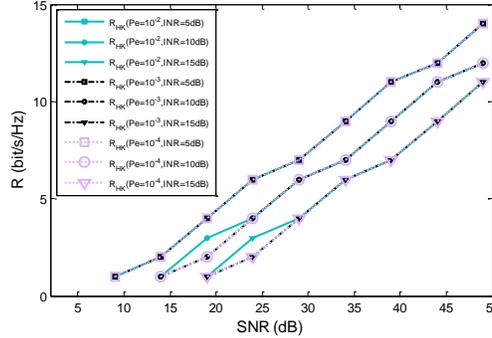


Figure 4. R_{HK} curves with $P_e=10^{-2}, 10^{-3}, 10^{-4}$

From Fig. 4, when P_e is large, R_{HK} curves with different P_e are different only when SNR differs INR within 15dB, while coincide with each other when SNR exceeding 30dB. This is because when P_e is variable, the difference of R_{HK} in Eq. (17) is reflected on the private bit information transmission rate R_{HK}^u as Eq. (15) and (16) shows. When SNR differs INR within 15dB, on the one hand, the ratio $\sqrt{P_0/P_1}$ is small, so δ_1 and δ_2 in Eq. (15) and (16) is small but not close to zero, with the increase of SNR, the first item $\left[\log \left(\frac{1 + \delta_k}{1 - \delta_k} \right)^2 \right]$ in Eq. (15) and (16) decreases but not close to zero also. On the other hand, due to the large P_e value of 10^{-2} , 10^{-3} and 10^{-4} , the corresponding $\Gamma(M, P_e)$ with three different P_e is small and very different from each other, so the second item

$\log\left(1 + \frac{P^u P_0}{(P^u P_1 + N_0)\Gamma(M^u, P_e)}\right)$ in Eq. (15) and (16) takes large value and very different from each other also. Therefore, the private bit transmission rate upper bound R_{HK}^u can not be determined particularly by certain one item in Eq. (15) or (16), R_{HK}^u will change with SNR and P_e simultaneously. Thus, the total information transmission rate upper bound R_{HK} will change with SNR and P_e when SNR differs INR within 15dB.

However, when SNR is much greater than INR for SNR exceeding 30dB, the ratio $\sqrt{P_0/P_1}$ is large, δ_1 and δ_2 in Eq. (15) and (16) is close to zero, so the first item $\left[\log\left(\frac{1+\delta_k}{1-\delta_k}\right)^2\right]$ in Eq. (15) and (16) tends to be zero, then the private bit transmission rate upper bound R_{HK}^u is mainly determined by the first item. While parameter P_e only affects the second item $\left[\log\left(1 + \frac{P^u P_0}{(P^u P_1 + N_0)\Gamma(M^u, P_e)}\right)\right]$ in Eq. (15) and (16). So the change of P_e will not change R_{HK}^u . The information transmission rate upper bound R_{HK} is completely the same with different P_e .

Decrease P_e further with 10^{-6} and 10^{-8} , the R_{HK} curves with both P_e values drawn by Eq. (17) are shown in Fig. 5.

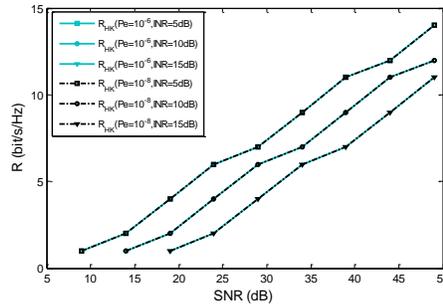


Figure 5. R_{HK} curves with P_e equals to 10^{-6} and 10^{-8}

From Fig. 5, when P_e values are small, R_{HK} curves with different P_e coincide completely with each other for all SNR and INR value. It means that the change of P_e dose not change the value of R_{HK} any more. This is mainly because that, when P_e is very small, the corresponding $\Gamma(M, P_e)$ is large and with great difference between different P_e . So the second item $\log\left(1 + \frac{P^u P_0}{(P^u P_1 + N_0)\Gamma(M^u, P_e)}\right)$ valued almost the same for a given SNR and INR. After the rounding function, $\left[\log\left(1 + \frac{P^u P_0}{(P^u P_1 + N_0)\Gamma(M^u, P_e)}\right)\right]$ is exactly the same integer value for different P_e . Thus, both R_{HK}^u and R_{HK}^w are only affected by SNR and INR, different system error probability P_e does not change the proposed information transmission rate upper bound R_{HK} .

5. Conclusions

A two-user M-QAM modulation symmetric interference system with HK scheme on weak interference region is studied. A constellation point minimal distance inequality method, which is more rational than direct swapping method, is proposed to derive the common bit transmission rate upper bound. Simulations found that: the proposed information transmission rate upper bound is smaller than

that in literature [11] only on low SNR region and can achieve two times the upper bound of two-user orthogonal system on high SNR region. The proposed information transmission rate upper bound stays the same with different system error probability when the system error probability is small, and differs within 1bits/s/Hz on low SNR region when the system error probability is large.

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7. References

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